Effect of flexible wall boundary in seismic design of liquid storage tanks with fluid structure interaction

Akash Rajan, Jayaraj Kochupillai
Advanced Dynamics and Control Lab
Dept. of Mech. Engg. College of Engg.
Trivandrum rajanakash@gmail.com
jayaraj@cet.ac.in

Manuel George, Anish R.
Dept. of Mech. Engg.
Saintgits College of Engg.
manuelgp89@gmail.com
anish.r@saintgits.org

***Abstract*— In this paper, the effect of fluid structure interaction on the modal characteristics of a cylindrical water tank without free surface effect is considered. Acoustic structure interaction using unsymmetric pressure based formulation is used to solve the coupled system using FEM and the procedure is validated using results from published literature. A 2D axisymmetric model is also presented to evaluate the same for axisymmetric mode shapes of the system and matched with the axisymmetric modes of the 3D tank model. The perturbation caused in the natural frequency of the coupled system is presented by comparing the coupled frequencies with the ordered uncoupled frequencies of the tank and fluid alone. Parametric study on the uncoupled frequencies of different tank configurations are also presented. The effect of flexible wall boundary condition on the convective and impulsive modes of tall and shallow aspect ratio tanks is shown and are compared to uncoupled structural modes. Parametric study is done for different fluid levels to characterize the dynamics of coupled system. Free surface is considered in fluid alone model to predict the slosh frequencies employing rigid wall boundary.**

***Keywords*— perturbation; slosh; convective and impulsive modes**

I. INTRODUCTION

The dynamic interaction between fluid and structure is a significant concern in many engineering problems. These problems include systems as diverse as off-shore and submerged structures, biomechanical systems, aircraft, suspension bridges, storage tanks. The interaction can drastically change the dynamic characteristics of the structure and consequently its response to transient and cyclic excitation. Therefore, it is desired to accurately model these diverse systems with the inclusion of Fluid Structure Interaction (FSI).

Fluid Structure Interaction (FSI) is defined as the interaction of some deformable or movable structure with an internal or surrounding flow. The important aspect of FSI is that there must be genuine interaction between the fluid and solid component. This implies that there is a transfer of energy from fluid to the solid and vice versa. Fluid structure interaction becomes particularly important when the liquid is almost incompressible and deformation on the solid cannot be neglected.

A literature survey has been conducted on seismic design of liquid storage tanks and its simulation. Most of the papers are having experimental evaluation of the structure with changes in shape and height. With the addition of another structure like roof on the tank also contribute significant effects on to it. Dynamic analysis of different shapes of tanks with varying quantities has been studied by various researchers. A. Ergin [6], introduced vibration problems by the interactions that take place between structure and fluid. This is due to the vibration of the structural surface in contact with the fluid medium imparting motion to the fluid, thus altering its pressure, and, hence, inducing reactive forces on its surface. In his investigation, it is assumed that the fluid is ideal, and fluid forces are associated with inertial effects only: namely, the fluid pressure on the wetted surface of the structure is in phase with the structural acceleration. M. Moslemi [12], bring up to identify the major parameters affecting the dynamic response of acoustic structures and to address the interaction between these parameters. Examined parameters include sloshing of liquid free surface, tank wall flexibility, vertical ground acceleration, tank aspect ratio, and base fixity. M. Amiri[11] introduced a series of ambient vibration tests on three tall liquid storage tanks with same height and different radius are considered, to determine the natural frequencies and, the modes of the vibration. Juan C. Virella[5] introduced the fundamental impulsive modes of vibration of cylindrical tank-liquid systems anchored to the foundation under horizontal motion. The roof and walls are represented with shell elements and the liquid is modeled using two techniques: the added mass formulation and acoustic finite elements. The literature survey gives a deep insight to the effects of various parameters on seismic structures and impact of different shapes on the natural frequency of the system, location of liquid storage tanks and the analysis on them.

The objective of the project is to evaluate the dynamic interaction between fluid and structure. The main reason for the study is better predictive Finite Element model development for design of structures which are prone to earthquake. By knowing the natural frequency and mode shapes, the structure can be better designed which are prone to common frequencies of earthquake. Parametric study is conducted by considering different wall flexibility, liquid level, tank dimensions etc. to characterize their significance in seismic response.

II. FINITE ELEMENT FORMULATION

A. *Abbreviations and Acronyms*

ρ_o : Mean fluid density
k : Bulk modulus of fluid
P : Acoustic pressure
t : Time
[M] : Structural mass matrix
[C] : Structural damping matrix
[K] : Structural stiffness matrix
{u},{u_e} : Nodal displacement vector
c : Speed of sound
{F_a} : Applied load vector
{n} : Unit normal to the interface S
{N} : Element shape function for pressure
{N'} : Element shape function for displacements
{P_e} : Nodal pressure vector
[R] : Coupling matrix
FSI : Fluid Structure Interaction
FEM : Finite Element Method

B. *Governing Equations and assumptions*

In acoustical fluid-structure interaction problems, the structural dynamics equation needs to be considered along with the Navier-Stokes equations. The same are simplified to get the acoustic wave equation using the following assumptions:

1. The fluid is compressible (density changes due to pressure variations).
2. The fluid is inviscid (no viscous dissipation).
3. There is no mean flow of the fluid.
4. The mean density and pressure are uniform throughout the fluid.
5. Analyses are limited to relatively small acoustic pressures so that the changes in density are small compared with the mean density.

The interaction of the fluid and the structure at the mesh interface causes the acoustic pressure to exert a force applied to the structure and the structural motions produce an effective "fluid load." The governing finite element matrix equations then become:

$$\begin{aligned}&\left[M_S\right]\left\{\ddot{U}\right\}+\left[K_S\right]\{U\}=\{F_S\}+[R]\{P\}\\&\left[M_f\right]\left\{\ddot{P}\right\}+\left[K_f\right]\{P\}=\left\{F_f\right\}-\rho_0[R]^T\left\{\ddot{U}\right\}\end{aligned} \quad (1)$$

[R] is a "coupling" matrix that represents the effective surface area associated with each node on the fluid-structure interface. It also takes into account the direction of the normal vector defined for each pair of coincident fluid and structural element faces that comprises the interface surface. The positive direction of the normal vector, as the ANSYS program uses it, is defined to be outward from the fluid mesh and in towards the structure.

Both the structural and fluid load quantities that are produced at the fluid-structure interface are functions of unknown nodal degrees of freedom. Placing these unknown "load" quantities on the left hand side of the equations and combining the two equations into a single equation produces the following:

$$\begin{bmatrix} M_S & 0 \\ \rho_0 R^T & M_f \end{bmatrix}\begin{Bmatrix}\ddot{U}\\ \ddot{P}\end{Bmatrix}+\begin{bmatrix} K_S & -R \\ 0 & K_f \end{bmatrix}\begin{Bmatrix}U\\ P\end{Bmatrix}=\begin{Bmatrix}F_S\\ F_f\end{Bmatrix} \quad (2)$$

The foregoing equation implies that nodes on a fluid-structure interface have both displacement and pressure degrees of freedom.

Where

$$\begin{aligned}&\left[M_f\right]=\int_V\frac{1}{c^2}\{N\}\{N\}^T dV\\&\left[K_f\right]=\int_V[B]^T[B]dV\\&\left[\mathrm{R}_e\right]=\int_s\{N\}\{n\}^T\{N'\}^T dS\end{aligned} \quad (3)$$

III. PROBLEM FORMULATION

Modal analysis is carried out in three different cases, **Case A:** axisymmetric 2D analysis of tall and shallow tank , **Case B**: 3D analysis is done for the same to compare with 2D and and **Case C:** fluid alone modes is analyzed to determine slosh frequencies.

A. *Case A*

In this study, two cylindrical concrete tank models with different aspect ratios, representative of two classes of tanks namely “Shallow” and “Tall” are investigated .The terms “Shallow” and “Tall” used in the current study have only relative meaning. The aspect ratio of the Tall tank model (H/D) is about three times higher than that of the Shallow tank model. The simplified geometries of the tanks are shown in figure 1. Dimensions of the tanks are selected such that the volume of stored water remains unchanged for both tank models [1]. The tanks are assumed to be anchored to the rigid ground such that no sliding or uplift may occur. Therefore, all base nodes located along the floor perimeter are fully restrained in all directions. As a result of this perfect anchorage assumption, no bending moment can be transferred from the wall to the floor and vice versa and therefore the tank floor may not be included in FE modelling of such containers. Since only anchored tanks are considered in this study, the tank floor is not modelled in FE simulation. Properties of the concrete are elastic Modulus=24.86GPa, Poisons ratio=0.16 and density=2400 kg/m3 and for water density =1000 kg/m3 and sonic velocity-1533m/s.

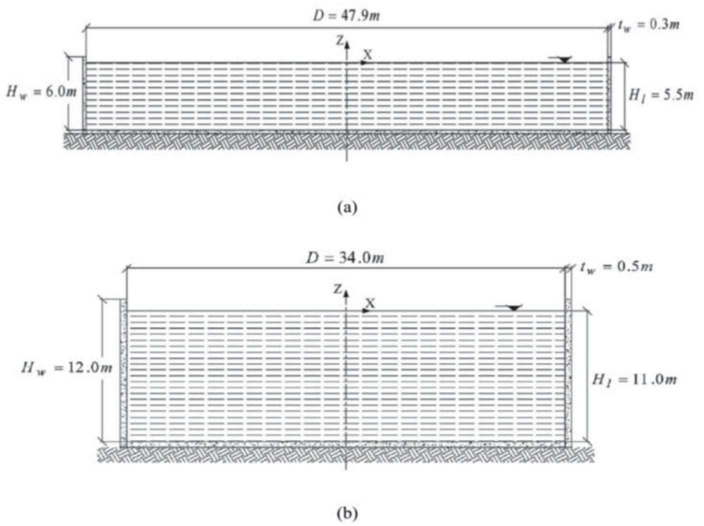


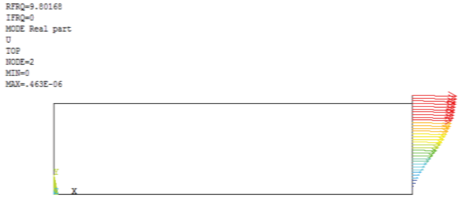
Figure 1: a) Shallow tank model b)Tall tank model [1]

Finite element models of tanks are constructed using the general purpose finite element package ANSYS and their dynamic parameters are derived from free vibration analyses. Modal analysis of 2D axisymmetric tank liquid systems is done in three stages. 1) Modal analysis of tank alone 2) Modal analysis of Fluid alone and 3) Modal analysis of tank with fluid. For axisymmetric analysis SHELL208 is a 2-node axisymmetric shell element with three degrees of freedom at each node(translations in the x and y directions and rotation about the z-axis) is used to model the tank. FLUID29 with four corner nodes and three degrees of freedom per node(translations in the nodal x and y directions and pressure at the interface for the structure present option) is used to model fluid . Only pressure DOF is present for structure absent option.

IV. RESULTS AND DISCUSSIONS

A. *Case A*

2D axisymmetric analysis of tank model to find the effect of coupling. Figure 1 shows the vector plot for the first mode shape of tank structure and fluid motion. Table shows effect of coupling modes. Modes 1,2 and 7 shows greater effective coupling resulting in higher frequency perturbation compared to uncoupled system (figure 3) .



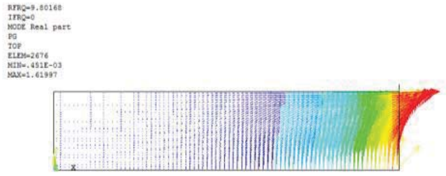


Figure 2:Ist mode shape of coupled system (vector plot)

TABLE I. COMPARISON OF 2D COUPLED FREQUENCY AND ORDERED UNCOUPLED FREQUENCY

Mode No:	Natural frequency in Hertz				
	Fluid alone	Structure alone	Ordered uncoupled[a]	Coupled (FSI)	Deviation
1	39.04	21.78	21.78 (s)	9.8	11.98
2	71.51	34.86	39.04 (f)	23.33	15.71
3	103.7	82.58	34.8 (s)	56.53	-21.73
4	136.06	136.02	71.51 (f)	73.62	-2.11
5	139.7	164.79	82.5 (s)	88.389	-5.889
6	145.1	286.66	103.7 (f)	108.05	-4.35
7	157	411.35	136.02 (s)	112.82	23.2

[a] *Values in brackets (s) denotes structure mode and (f) denote fluid mode*

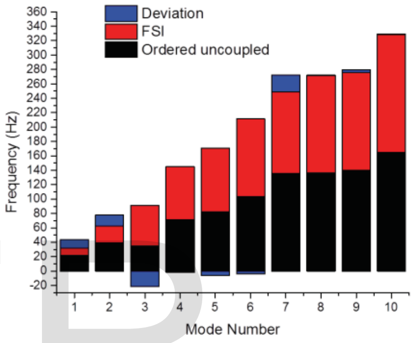


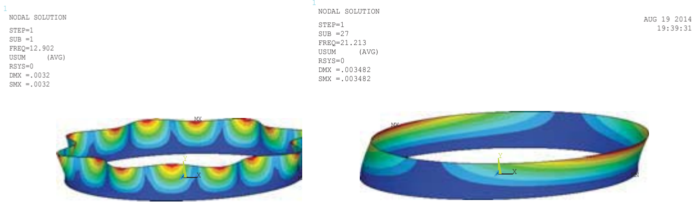
Figure 3: Graph showing perturbation in frequency of coupled and ordered uncoupled frequency

Due to added mass effect, coupled system frequency for mode 1 has reduced from 21.78 Hz to 9.8 Hz. Computed eigen frequencies (Hz) of the coupled and uncoupled system for fluid in concrete cavity is given in table 1, New fluid dominated modes are formed due to coupling.

The magnitude of structure density and fluid density are comparable (double). Due to the fact the eigenvalues of the coupled problem are having small perturbation than that of uncoupled problem. The frequency reduction is found to be maximum for the modes with greater pressure gradient at the fluid structure interface.

B. *Case B*

The tank structure is modelled using four noded isoparametric quadrilateral SHELL 181 element. The element has four nodes with six degrees of freedom at each node: translations in the x, y and z axes and corresponding three rotations and the fluid domain is modelled using Fluid 30 element having eight corner nodes with four degrees of freedom per node: translations in the nodal x, y and z directions and pressure at the interface for the structure present option. Only pressure DOF is present for structure absent option.. Three dimensional finite element models of Shallow and Tall tanks are constructed using the finite element package ANSYS and their fundamental natural frequencies are derived from modal or free vibration analyses. The results of the analyses are presented in this section. There are two types of modes: impulsive and



shape of shallow tank

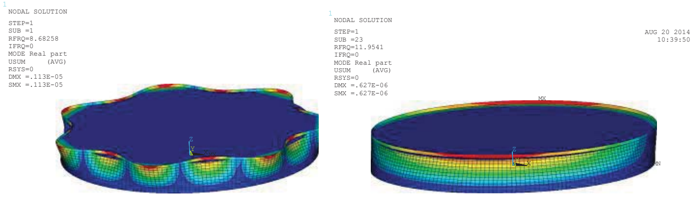
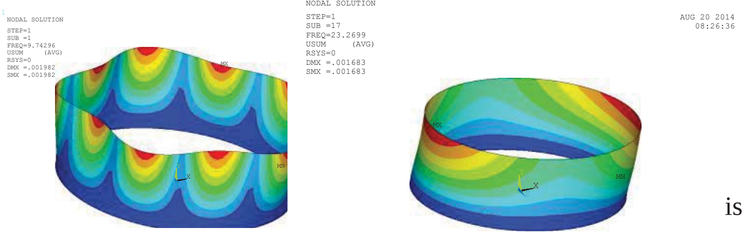


Figure 5:first convective and firs



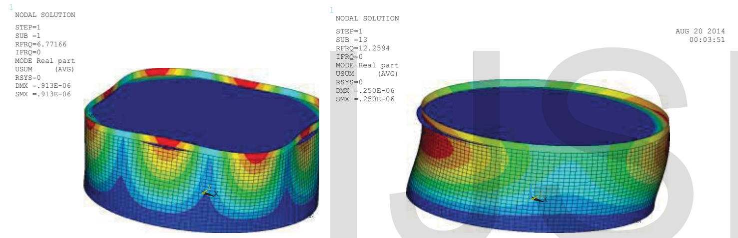


Figure 7:first convective and first impulsive mode shape of tall tank with fluid

Convective modes or shell modes of the tank is dominated by circumferential deformation while the impulsive mode or beam type modes are dominated by the axial deformation of the tank. The two modes (I^{st} frequencies) are found for different water heights and are tabulated

Figure 4 show first convective and impulsive modes of shallow tank without fluid and figure 5 shows the same for shallow tank with fluid. When fluid is loaded to shallow tank impulsive mode frequency got reduced from 21.21 to 11.96 Hz and convective mode frequency got reduced from 12.83 to 8.68 Hz. The rate of decrease is more for impulsive mode due to greater participation of fluid mass.

Figure 6 shows first convective and impulsive modes of tall tank without fluid and figure 7 shows that of tall tank with fluid. When fluid is loaded to tall tank impulsive mode frequency got reduced from 23.26 to 12.26 Hz and convective mode frequency got reduced from 9.74 to 6.77 Hz.

TABLE II. IMPULSIVE VS CONVECTIVE MODES OF A SAHLLOW TANK WITHOUT FLUID

Tank height in m	**Frequency in Hertz for shallow tank**	
	First Convective mode	***First Impulsive mode***
1	150.38	150.22
2	44.65	44.368
3	26.61	27.47
4	19.54	23.28
5	15.47	21.86
6	**12.83**	**21.21**
8	9.61	20.40

[b.] Values in bold letters denote the actual water tank under test

TABLE III. IMPULSIVE VS CONVECTIVE MODES OF A SAHLLOW TANK WITH FSI

Water height in m	**Frequency in Hertz for shallow tank**	
	First Convective mode	***First Impulsive mode***
1	13.8	21.46
2	14.42	21.54
2.5	14.396	21.13
3	13.93	19.88
4	11.9	16.17
5	9.64	13.169
5.5	**8.68**	**11.96**

[c.] Values in bold letters denote the actual water tank under test

Structure alone impulsive mode frequency for shallow tank is 21.31Hz and for flexible (FSI) case it is 21.463

TABLE IV. IMPULSIVE VS CONVECTIVE MODES OF A TALL TANK WITHOUT FLUID

tank height in m	**Frequency in Hertz for tall tank**	
	First Convective mode	***First Impulsive mode***
1	230.32	228.27
3	40.75	41.33
5	23.41	30.76
7	16.53	28.24
9	12.76	26.34
11	10.514	24.32
12	**9.74**	**23.26**
16	7.04	19.3

[d.] Values in bold letters denote the actual water tank under test

TABLE V. IMPULSIVE VS CONVECTIVE MODES OF A TALL TANK WITH FSI

Water height in m	**Frequency in Hertz for tall tank**	
	First Convective mode	***First Impulsive mode***
1	10.062	23.77
2	10.35	24.23
3	10.602	24.53
4	10.78	24.1
5	10.715	21.77
6	10.343	19.19
9	8.17	14.24
11	**6.77**	**12.26**

[e.] Values in bold letters denote the actual water tank under test

It is found that at lower fluid levels impulsive frequency increase is observed which is due to stiffening of tank wall structure. When the water height is gradually increased this impulsive frequency is reduced. At fluid levels greater than half of the tank height, greater frequency reduction is observed due to greater fluid mass participation. Thus at lower levels of fluid, impulsive modes are above the seismic region and it become less dangerous. It is clear from this parametric study that the effect of fluid level on the impulsive mode, which is the most significant design criterion for seismic resistant structures, is found to be critical. Hence the fluid content in the tank must always be maintained at a safe level as per the design. From the frequency decrease rate we can conclude that the fluid level in the case of tall tank is more critical than the case of shallow tank due to the higher frequency decrease.

$$\Delta f_{\text{convective(shallow)}} > \Delta f_{\text{convective(tall)}}$$

$$\Delta f_{\text{impulsive(tall)}} > \Delta f_{\text{impulsive(shallow)}}$$

Where ,

Δf = change in frequency

C. Case C

Free surface is considered in fluid with rigid walls to predict the slosh frequencies. Pressure plot of the slosh mode shapes considered are shown in figure 8 and 9.. The variation in frequency with fluid height for the fundamental slosh mode is shown in figure 10. It is found that the rate of increase in the frequency is more in the beginning and leads to a saturation with grater fluid levels. This results matches with the frequency computed using the standard design formulas [1, 3].

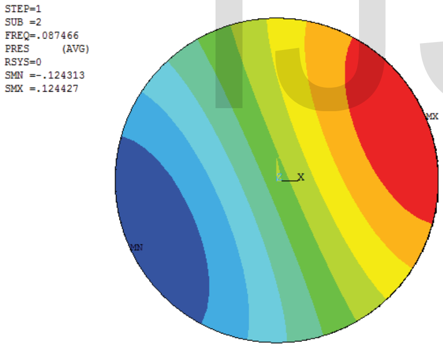


Figure 8:first slosh mode of the fluid with rigid wall

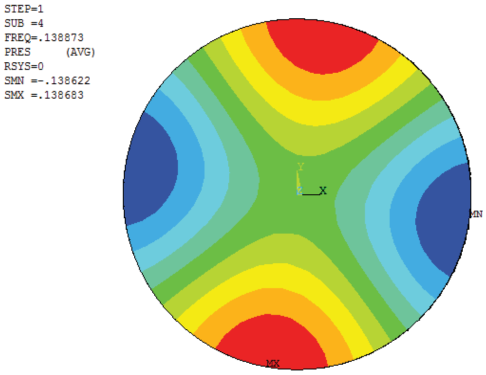


Figure 9:second slosh mode of the fluid with rigid wall

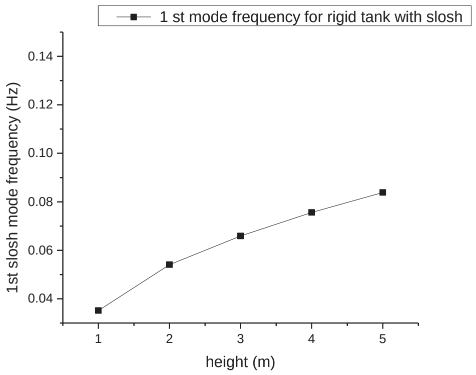


Figure 10

At h=1 slosh frequency is below .04Hz and h=5 it is increased upto 0.09Hz. It is found that first slosh mode frequency got reduced when fluid height is reduced. It can be compared with practical example of a beaker with fluid. When the beaker is full of water it is easy to handle the beaker without sloshing than when it is at lower level. Hence it is seen that fluid at lower levels slosh at very low frequency.

V. CONCLUSION

The effect of fluid structure interaction on the modal characteristics of a cylindrical water tank without free surface effect is presented with parametric study on fluid height and diameter of tank. A 2D axisymmetric model is evaluated for finding axisymmetric mode shapes of the system and is compared with 3D. And it is observed that mode shapes are matching. The rate of decrease in frequency is more for the impulsive mode due to the participation of fluid mass. Change in convective frequency for shallow tank is greater than tall tank. But change in impulsive frequency is observed greater for tall tank than shallow tank. Among the tall and shallow tank models greater impulsive frequency is observed for tall tank without fluid, but the rate of decrease of the same with coupled system is more for tall tank and hence the criticality of water level is more significant.

Acknowledgment

This work was supported by Centre for Engineering Research and Development, Trivandrum, Kerala and All India Council for Technical Education [RPS grant No: 8023/RID/RPS-24/2011-12].

References

[1] G M. Moslemi, M.R. Kianoush. Parametric study on dynamic behaviour of cylindrical ground supported tanks. *Engineering Structures 42* (2012): 214-230.

[2] Jean Francois Sigrist, Stephane Garreau. Dynamic analysis of fluid – structure interaction problems with modal methods using pressure based fluid finite elements. *Fininte elements in analysis and design 43* (2007): 287-300.

[3] GSDMA Guidelines for seismic design of liquid storage tanks prepared by National Information Centre of Earthquake Engineering, *Indian Institute of Technology Kanpur with funding by Gujarat state disaster management authority Gandhinagar* (2007).

[4] J.R Cho, J.M Song, J.K Lee. Finite element techniques for the free-vibration and seismic analysis of liquid-storage tanks. *Finite element in analysis and design 37* (2001): 467-483.

[5] Juan C Virella, Luis A Godoy, Luiz E Suarez. Fundamental modes of tank-liquid systems under horizontal motion. *Engineering structures 28* (2006): 1450-1461.

[6] A.Ergin, P. Temarel; 2002, "Free vibration of a partially liquid-filled and Submerged, horizontal cylindrical shell" Journal of Sound and Vibration 254(5), PP 951-965

[7] Ahmed A Elshafey, Mahmoud R Haddara, H. Marzouk. Dynamic response of offshore jacket structures under random loads. *Marine structures 22* (2009): 504-521.

[8] Jun Zheng Chen. Generalized SDOF system for dynamic analysis of concrete rectangular liquid storage tank systems. *Ryerson University, Toronto* (2010)

[9] Amirreza Ghaemmaghami. Dynamic time history response of concrete rectangular liquid storage tanks. *Ryerson University, Toronto* (2010)

[10] Alemdar Bayraktar, Baris Sevim, Temel Turker. Effect of the model updating on the earthquake behaviour of steel storage tanks. *Journal of constructional steel research 66* (2010): 462-469.

[11] O. Curadelli, D. Ambrosini, A. Mirasso, M. Amani. Resonant frequencies in an elevated spherical container partially filled with water: *FEM and measurement. Journal of fluids and structures 26* (2010): 148-159.

[12] M. Amiri, S.R Sabbagh-Yezdi. Ambient vibration test and finite element modelling of tall liquid. *Thin-walled structures 49* (2011): 974-983.

[13] M. Moslemi, M.R Kianoush, W. Pogorzelski. Seismic response of liquid-filled elevated tanks. *Engineering structures 33* (2011): 2074-2084.